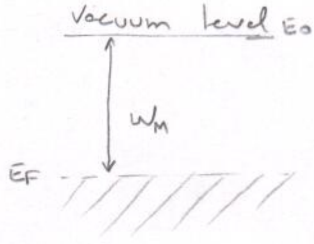
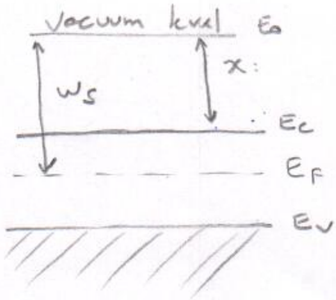
Work functions:

- metals : W_M : minimum energy to extract an electron from a solid to vacuum
- semiconductors : W_S is the work function $(E_0 - E_F)$ and χ is the electron affinity $(E_0 - E_c)$

metal :



semiconductor:



- Vacuum level is the reference level.

$$\phi_{bi} = W_{Sp} - W_{Sn} = (E_C - E_F)_{x \ll 0} - (E_C - E_F)_{x \gg 0}$$

$$= KT \ln \frac{N_C}{n_0(x \ll 0)} - KT \ln \frac{N_C}{n_0(x \gg 0)} =$$

$$= KT \ln \frac{n_0(x \gg 0)}{n_0(x \ll 0)}$$

$n_0(x \gg 0)$ → electron are majority carriers $n_0 = N_D$

$n_0(x \ll 0)$ → electron are minority carriers $n_0 = n_i^2 / N_A$

- since χ and band gap is the same
- since : $n = N_C \cdot \exp\left(\frac{E_F - E_C}{KT}\right)$

$$\boxed{\phi_{bi} = KT \ln \frac{N_D \cdot N_A}{n_i^2}}$$

for which we assumed :
- extrinsic semiconductors $N_D \cdot N_A \gg n_i$
- all dopants ionized

of course, if $\phi_{bi} = 0$ ⇒ semiconductor is intrinsic so no ϕ_{bi}!

P-N junction:

Depletion approximation:
$\begin{cases} \cdot \text{ QNR are perfectly neutral} \Rightarrow \rho = 0 \text{ and } \frac{\partial E}{\partial x} = 0 \\ \cdot \text{ Dopants are ionized} \end{cases}$ $\quad E = 0$

QNR $\quad \rho_0(x) = 0$, in p-QNR

$$\text{SCR} \begin{cases} \rho_0(x) \simeq -q \cdot N_A , & -x_p < x < 0 \\ \rho_0(x) \simeq +q \cdot N_D , & 0 < x < x_n \end{cases}$$

QNR $\quad \rho_0(x) = 0$, in n-QNR

1) **Electric field:** $\quad \frac{dE}{dx} = \frac{\rho(x)}{\varepsilon}$

QNR : $E(x) = cte = 0$

$$\text{SCR} \begin{cases} E(x) = -\frac{q \cdot N_A}{\varepsilon} x + A = -q \cdot \frac{N_A}{\varepsilon}(x + x_p) , & -x_p < x < 0 \\ E(x) = +\frac{q \cdot N_D}{\varepsilon} \cdot x + B = +q \cdot \frac{N_D}{\varepsilon}(x - x_n) , & 0 < x < x_n \end{cases}$$

QNR : $E(x) = cte = 0$

Since E is continuous:
$E(x = -x_p) = 0$
$E(x = x_n) = 0$

2) **Electrostatic potential:** $\quad E = -\frac{d\Phi}{dx}$, one more integration:

$$\text{SCR:} \begin{cases} \Phi(x) = +q \cdot \frac{N_A}{2\varepsilon} \cdot (x^2 + 2x_p \cdot x) , & -x_p < x < 0 \\ \Phi(x) = -q \frac{N_D}{2\varepsilon} \cdot (x^2 - 2x_n \cdot x) , & 0 < x < x_n \end{cases}$$

QNR : $d\Phi/dx = 0 \Rightarrow \Phi$ must be constant!

$$x = -x_p \Rightarrow \Phi(x = -x_p) = q \cdot \frac{N_A}{2\varepsilon} \cdot (-x_p^2) = -q \frac{N_A \cdot x_p^2}{2\varepsilon} , \text{ in p-QNR}$$

$$x = +x_n \Rightarrow \Phi(x = +x_n) = -q \cdot \frac{N_D}{2\varepsilon} \cdot (-x_n^2) = +q \frac{N_D \ x_n^2}{2\varepsilon} , \text{ in n-QNR}$$

#4

Using: 1) charge neutrality : $q \cdot N_A \cdot x_p = q \cdot N_D \cdot x_n \Rightarrow x_p = \frac{\cancel{q} N_D x_n}{\cancel{q} N_A}$

2) $\phi(x_n) - \phi(-x_p) = \phi_{bi}$

thus:

$$\phi(x_n) - \phi(-x_p) = \frac{q \cdot N_D \cdot x_n^2}{2\varepsilon} + \frac{q \cdot N_A \cdot x_p^2}{2\varepsilon} = \phi_{bi}$$

$$\frac{q}{2\varepsilon}\left(N_D x_n^2 + \cancel{N_A} \cdot \frac{N_D^2}{N_A^{\cancel{2}}} \cdot x_n^2\right) = \phi_{bi}$$

$$\frac{q \cdot x_n^2}{2\varepsilon} \cdot \frac{N_D N_A + N_D^2}{N_A} = \phi_{bi}$$

$$\therefore\ x_n = \sqrt{\frac{2 \cdot \varepsilon \cdot \phi_{bi} \cdot N_A}{q \cdot N_D (N_A + N_D)}}$$

$$x_p = \sqrt{\frac{2 \cdot \varepsilon \cdot \phi_{bi} \cdot N_D}{q \cdot N_A (N_D + N_A)}}$$

$$x_{SCR} = x_n + x_p$$

$$x_{SCR} = \sqrt{\frac{2\varepsilon (N_D + N_A)\phi_{bi}}{q \cdot N_A N_D}}$$

the maximum electric field :

$$\mathcal{E}(x=0) = -\frac{q \cdot N_A}{\varepsilon} \cdot x_p$$

$$= -\frac{q \cdot N_D}{\varepsilon} \cdot x_n$$

Due to continuity of $\vec{E}$ we get charge neutrality equation : $q N_a x_p = q N_D x_n$

$$\therefore\ \mathcal{E}_{max} = -\sqrt{\frac{2q \cdot N_A N_D \cdot \phi_{bi}}{\varepsilon (N_D + N_A)}}$$

Exercise : #5

$N_A = 5 \times 10^{18}\ cm^{-3}$

$N_D = 1 \times 10^{17}\ cm^{-3}$

$\downarrow$

$N_A = 10^{16}\ cm^{-3}$

(a) $\phi_{bi} = \frac{kT}{q} \cdot \ln \frac{N_D N_A}{n_i^2} = 0.026 \ln \frac{5.10^{18} cm^{-3}\ 10^{17} cm^{-3}}{(1 \times 10^{10} cm^{-3})^2} = 0.94 V$

(b) $x_p = \sqrt{\frac{2\varepsilon N_D \phi_{bi}}{q N_A (N_D + N_A)}} = \sqrt{\frac{2 \times 11.7 \times 8.85 \times 10^{-14} F/cm \times 10^{17} cm^{-3} \times 0.94 V}{1.6 \times 10^{-19} C \times 5.10^{18} cm^{-3} \cdot (10^{17} + 5.10^{18} cm^{-3})}}$

$= 2.2\ nm$

(c) $x_n = x_p \cdot \frac{N_A}{N_D} = \frac{2.2\ nm \times 5.10^{18} cm^{-3}}{10^{17} cm^{-3}} = 110\ nm \longrightarrow 1100\ nm$ if $N_D \to 10^{16}$ Dominates

* Depletion region extends more over less doped regions.

d)

$$|E_{max}| = \frac{q \cdot N_D \cdot x_n}{\varepsilon} = \frac{1.6 \times 10^{-19} C \times 10^{16} cm^{-3} \times 1.1 \times 10^{-5} cm}{11.7 \times 8.85 \times 10^{-14} F/cm} = 16.8 \times 10^{4} V/cm$$

$$|E_{max}| = 168 \, kV/cm$$

Critical Electric field of Silicon :

$$E_{br} = 300 \, kV/cm$$

Under bias V:

$E = -q \cdot V$ ⇒ by applying a voltage V we are giving potential energy to electrons

$$q\phi_{bi} \longrightarrow q\phi_{bi} - qV$$

$$x_{SCR}(V) = \sqrt{\frac{2\varepsilon (N_D + N_A)\,(\phi_{bi} - V)}{q \cdot N_A N_D}} = x_{SCR}(V=0) \cdot \sqrt{1 - \frac{V}{\phi_{bi}}}$$

$$|E_{max}(V)| = \sqrt{\frac{2q \cdot N_A N_D \cdot (\phi_{bi} - V)}{\varepsilon\,(N_A + N_D)}} = |E_{max}(V=0)| \cdot \sqrt{1 - \frac{V}{\phi_{bi}}}$$

$E_{cr} = 300\ kV/cm$

$$\Rightarrow \quad 300\,\frac{kV}{cm} = 168\,\frac{kV}{cm} \cdot \sqrt{1 - \frac{V}{0.84}}$$

$$\boxed{V_{br} = -2\ V}$$

IV curve quantitative discussion:

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forward bias: $\downarrow \vec{E}$ ⇒ $\downarrow$ drift thus diffusion prevails
$\bar{e}$ flows from n- to p-side

Reverse bias: $\uparrow \vec{E}$ ⇒

$$\boxed{J_e = \mu_e \cdot n \cdot \frac{dE_{fe}}{dx}}$$

$$J_e(-x_p) \simeq -q \cdot v_e^{diff}(-x_p) \cdot n'(-x_p)$$

$$J_h(x_n) \simeq q \cdot v_h^{diff}(+x_n) \cdot p'(+x_n)$$

these terms are the minority carrier current injected in each QNR.

In T.E:

$$\phi_{bi} = \phi_0(x_n) - \phi_0(-x_p) = \frac{kT}{q} \ln\left(\frac{n_0(x_n)}{n_0(-x_p)}\right) \Rightarrow \phi_{bi} = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right) \quad (1)$$ #7

Under low level injection (LLi), quasi-equilibrium:

$$\phi(x_n) - \phi(-x_p) = \frac{kT}{q} \ln\left(\frac{n(x_n)}{n(-x_p)}\right) \simeq \phi_{bi} - V$$

$$n(x_n) \simeq N_D$$

So:

$$\frac{N_D}{n(-x_p)} = \exp\left(\frac{q(\phi_{bi} - V)}{kT}\right)$$

$$n(-x_p) = \underbrace{N_D \cdot \exp\left(-\frac{q\,\phi_{bi}}{kT}\right)}_{\frac{n_i^2}{N_A} \text{ (from (1))}} \cdot \exp\left(+\frac{qV}{kT}\right)$$

thus:

$$n(-x_p) = \frac{n_i^2}{N_A} \cdot \exp\left(+\frac{qV}{kT}\right)$$

In terms of excesses:

in T.E: $n_0(-x_p) = \dfrac{n_i^2}{N_A}$

under L.L.I: $n(-x_p) = \dfrac{n_i^2}{N_A} \exp\left(+\dfrac{qV}{kT}\right)$

excess $n' = n(-x_p) - n_0(-x_p)$

$$\boxed{n' = \frac{n_i^2}{N_A} \cdot \left(\exp\left(\frac{qV}{kT}\right) - 1\right)}$$

#8

Minority carriers at the edge of the SCR diffuse inside the QNRs

excess concentrations:

$$\begin{cases} p'(x) = p'(x_n) \cdot \exp\left(\dfrac{-x + x_n}{L_h}\right), & x \geq x_n \\ n'(x) = n'(-x_p) \cdot \exp\left(\dfrac{x + x_p}{L_n}\right), & x \leq -x_p \end{cases}$$

where

Diffusion lengths

$$\begin{cases} L_h = \sqrt{D_h \cdot \tau} \\ L_n = \sqrt{D_n \cdot \tau} \end{cases}$$

D_n: Diffusion coefficient (cm^2/s)

τ: carrier lifetime (s)

And $$v_e^{diff}(-x_p) = -\frac{D_e}{L_e} \quad : \quad \frac{[cm^2/s]}{[cm]}$$

$$v_h^{diff}(+x_n) = +\frac{D_h}{L_h}$$

diffusion coefficient

diffusion length

thus $$J_e(-x_p) = q \cdot \frac{n_i^2}{N_A} \cdot \frac{D_e}{L_e} \cdot \left(\exp\frac{qV}{kT} - 1\right)$$

$$J_h(x_n) = q \cdot \frac{n_i^2}{N_D} \cdot \frac{D_h}{L_h} \cdot \left(\exp\frac{qV}{kT} - 1\right)$$

$$\therefore \boxed{J = J_s \cdot \left(\exp\frac{qV}{kT} - 1\right)}$$